Volatility Reduction in Electricity Markets via Centralized Call Options

Working paper

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Abstract

Increased penetration of wind energy will make electricity market prices more volatile. As a result, market participants will bear increased financial risks. In this paper, we propose a centralized market for cash-settled call options. A call option for electricity entitles its holder the right to claim a monetary reward equal to the positive difference between the realtime price and a pre-negotiated strike price in exchange for an upfront fee. Such options can reduce payment volatilities. We analytically characterize the outcomes of the option trade over a copperplate power system example, and numerically explore the same for a modified IEEE 14-bus test system.

1 Introduction

Wind energy is uncertain (difficult to forecast), intermittent (shows large ramps), and largely uncontrollable (output cannot be altered on command). They fundamentally differ from dispatchable generation that "can be controlled by the system operator and can be turned on and off based primarily on their economic attractiveness at every point in time" [1]. It has been widely recognized that escalated penetration of wind will dampen electricity prices. Wind is a (near) zero marginal cost resource, and hence, alters the merit-order of generators at the base of the supply stack, and not at the margin. "Free" wind shifts the offer stack to the right, leading to the intersection with the demand curve at a lower price level. Empirical evidence corroborates that hypothesis. For example, see the analysis by Ketterer [2] for the German market, Munksgaard and Morthorst [3] for the Danish market, and de Miera et al. [4] in the Spanish electricity market, among others. Green's model-oriented analysis [5] for the British market resonates the same sentiments.

A perhaps less studied effect of large-scale wind integration is its contribution to price volatility. Dispatchable (and often marginal) generators need to compensate for variations in wind availability, leading to variations in energy prices. Market analysis concurs with that conclusion, e.g., see the studies by Woo et al. [6] for ERCOT, Martinez-Anido [7] for New England, Jónsson et al. [8] for the Danish market, and Ketterer [2] for the German one.[1] Gerasimova [10], studying the Nord pool (Finland, Sweden, Norway, Denmark), shows that intraday price variations in parts of Finland and Sweden – measured in terms of the expected difference in daily on-peak and off-peak prices – have roughly doubled in 2008-2016 from that in 2000-2007. Such trends are likely to persist and perhaps grow, given the rapid growth in wind penetration.

How can market participants hedge against financial risks from these price variations? Financial instruments such as forwards, futures, swaps, and options can help mitigate such risks; see [11–13] for their use in electricity markets. The focus of the current paper is on the use of cash-settled call options. The holder of one unit of such a call option entitles the buyer to receive a cash payment equal to the real-time price of electricity less the negotiated strike price, in exchange for an upfront fee. Options are typically traded bilaterally. In contrast, we propose a *central clearing mechanism for call options*, and demonstrate how it can effectively reduces payment volatilities for electricity market participants.

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[1] Price variations differ considerably across a day; they are positively correlated with demand, as shown in [5] using data from the British market. They also exhibit seasonal variations. These variations tend to be greater in the summer, as the Australian market analysis in [9] reveals.

Organization of this paper

Section 1 introduces generic notation used throughout. Section 2 presents a dispatch and pricing model for a two-period electricity market. Then, Section 3 motivates the use of call options through the study of a bilateral call option trade between a wind power producer and a dispatchable peaker power plant in a copperplate power system example. Recognizing that engaging in multiple bilateral trades can be challenging for participants on a daily basis (and might lead to low liquidity in these markets), we propose a centralized clearing mechanism for call options in Section 4. We illustrate how it generalizes the bilateral trade in the single-bus power system example in Section 5 and conduct numerical experiments on the IEEE 14-bus test system [14] in Section 6 using our open-source market clearing tool in [15]. The paper concludes in Section 7. Proofs of all results can be found in the Appendix (Section 8).

Notation

We let $\mathbf{R}$ denote the set of real numbers, and $\mathbf{R}_+$ (resp. $\mathbf{R}_{++}$) denote the set of nonnegative (resp. positive) numbers. For $z \in \mathbf{R}$, we let $z^+ := \max\{z, 0\}$. For a random variable Z, we denote its expectation by $\mathbb{E}[Z]$, its variance by $\mathsf{var}[Z]$, and its cross-covariance with another random variable X by $\mathsf{cov}(X,Z)$; note that $\mathsf{cov}(X,X) = \mathsf{var}[X]$. For an event $\mathscr{E}$, we denote its probability by $\mathbb{P}\{\mathscr{E}\}$ for a suitably defined probability measure $\mathbb{P}$. The indicator function for an event $\mathscr{E}$ is given by $\mathbb{1}_{\{\mathscr{E}\}}$. In any optimization problem, a decision variable x at optimality is denoted by x^*.

2 Describing the marketplace

An organized wholesale electricity market is comprised of consumers and producers of electricity. The consumers are utility companies or retail aggregators who represent a collection of retail customers. In this work, we consider two types of producers – dispatchable generators and variable renewable wind power producers. Dispatchable generators can alter their power output within their capabilities on command, e.g., nuclear, coal, natural gas, biomass or hydro power based power plants. In contrast, the available production capacity of variable producers rely on an intermittent resource like wind and solar energy. The system operator (denoted SO) implements a centralized market mechanism to balance demand and supply of power within the network constraints.

Modeling uncertainty in supply

We model a two-period market model as follows. Identify $t = 0$ as the ex-ante stage, prior to the uncertainty being realized (which can be viewed as the *day-ahead stage* in current markets), and $t = 1$, the ex-post stage (which can be viewed as the *day-ahead stage*). Let $(\Omega, \mathscr{F}, \mathbb{P})$ denote the probability space, describing the uncertainty. Here, Ω is the collection of possible scenarios at $t = 1$, $\mathscr{F}$ is a suitable σ-algebra over Ω, and $\mathbb{P}$ is a probability distribution over Ω. We assume that all market participants know $\mathbb{P}$. Also, assume that Ω is compact.

Modeling the market participants

Let d denote the aggregate inflexible demand that is accurately known a day in advance.[2] Let $\mathfrak{G}$ and $\mathfrak{R}$ denote the collection of dispatchable generators and variable renewable wind power producers, respectively. We model their individual capabilities as follows.

- Let each dispatchable generator $g \in \mathfrak{G}$ produce x_g^ω in scenario $\omega \in \Omega$. We model its ramping capability by letting $|x_g^\omega - x_g^0| \le \ell_g$, where x_g^0 is a generator set point that is decided ex-ante, and ℓ_g is the ramping limit. Let the installed capacity of generator g be x_g^{cap}, and hence $x_g^\omega \in [0, x_g^{\mathsf{cap}}]$. Its cost of production is given by the smooth convex increasing map $c_g : [0, x_g^{\mathsf{cap}}] \to \mathbf{R}_+$.

[2] Day-ahead demand forecasts in practice are typically quite accurate. Notwithstanding the availability of such forecasts, our work can be extended to account for demand uncertainties.

- Each variable renewable wind power producer $r \in \mathfrak{R}$ produces x_r^ω in scenario $\omega \in \Omega$. It has no ramping limitations, but its available production capacity is random, and we have $x_r^\omega \in [0, \bar{x}_r^\omega] \subseteq [0, x_r^{\text{cap}}]$. That is, $\bar{x}_r^\omega$ denotes the random available capacity of production, and x_r^{cap} denotes the installed capacity for r. Similar to a dispatchable generator, the cost of production for r is given by the smooth convex increasing map $c_r : [0, x_r^{\text{cap}}] \to \mathbf{R}_+$.

We call a vector comprised of x_g for each $g \in \mathfrak{G}$ and x_r for each $r \in \mathfrak{R}$ a *dispatch*.

The SO decides the dispatch decisions and the compensations of all market participants. In the remainder of this section, we describe the so-called conventional dispatch and pricing model, that serves as a useful benchmark for electricity market designs under uncertainty, e.g., in [16, 17].

Conventional dispatch and pricing model

We assume that the SO knows c_g, x_g^{cap} for each $g \in \mathfrak{G}$ and $x_r, x_r^{\text{cap}}, \bar{x}_r$ for each $r \in \mathfrak{R}$. In practice, the cost functions are derived from supply offers from the generators. The market participants, in general, may have incentives to misrepresent their cost functions. Analyzing the effects of such strategic behavior is beyond the scope of this paper.

The day-ahead stage

The SO computes a forward dispatch against a point forecast of all uncertain parameters. In particular, the SO replaces the random available capacity $\bar{x}_r^\omega$ by a certainty surrogate $\bar{x}_r^{\text{CE}} \in [0, \bar{x}_r^{\text{cap}}]$ for each $r \in \mathfrak{R}$. A popular surrogate[3] is given by $\bar{x}_r^{\text{CE}} := \mathbb{E}[\bar{x}_r^\omega]$. The forward dispatch is found by solving

$$
\begin{aligned}
\text{minimize} \quad & \sum_{g \in \mathfrak{G}} c_g(X_g) + \sum_{r \in \mathfrak{R}} c_r(X_r), \\
\text{subject to} \quad & \sum_{g \in \mathfrak{G}} X_g + \sum_{r \in \mathfrak{R}} X_r = d, \\
& X_g \in [0, x_g^{\text{cap}}], \ \ X_r \in [0, \bar{x}_r^{\text{CE}}], \\
& \text{for each } g \in \mathfrak{G}, \ r \in \mathfrak{R},
\end{aligned}
$$

over $X_g \in \mathbf{R}, g \in \mathfrak{G}$, and $X_r \in \mathbf{R}, r \in \mathfrak{R}$.

The forward price is given by the optimal Lagrange multiplier of the energy balance constraint. Denoting this price by P^*, generator $g \in \mathfrak{G}$ is paid $P^* X_g^*$, while producer $r \in \mathfrak{R}$ is paid $P^* X_r^*$. Aggregate consumer pays $P^* d$.

At real-time

Scenario ω is realized, and the SO solves

$$
\begin{aligned}
\text{minimize} \quad & \sum_{g \in \mathfrak{G}} c_g(x_g^\omega) + \sum_{r \in \mathfrak{R}} c_r(x_r^\omega), \\
\text{subject to} \quad & \sum_{g \in \mathfrak{G}} x_g^\omega + \sum_{r \in \mathfrak{R}} x_r^\omega = d, \\
& x_g^\omega \in [0, x_g^{\text{cap}}], |x_g^\omega - X_g^*| \leq \ell_g, \\
& x_r^\omega \in [0, \bar{x}^\omega], \ \ \text{for each } g \in \mathfrak{G}, \ r \in \mathfrak{R},
\end{aligned}
$$

over $x_g^\omega \in \mathbf{R}, g \in \mathfrak{G}$ and $x_r^\omega \in \mathbf{R}, r \in \mathbf{R}$. The real-time (or spot) price is again defined by the optimal Lagrange multiplier of the energy balance constraint, and is denoted by $p^{\omega,*} \in \mathbf{R}_+$. Note that the optimal X_g^* computed at $t = 0$ defines the generator set-points x_g^0 for each generator $g \in \mathfrak{G}$. Generator $g \in \mathfrak{G}$ is paid $p^{\omega,*}\left(x_g^{\omega,*} - X_g^*\right)$, while producer $r \in \mathfrak{R}$ is paid $p^{\omega,*}\left(x_r^{\omega,*} - X_r^*\right)$. The aggregate consumer does not have any real-time payments, since there is no deviation in the demand.

The total payments to each participant is the sum of her forward and real-time payments. Call these payments π_g^ω for each $g \in \mathfrak{G}$ and π_r^ω for each $r \in \mathfrak{R}$ in scenario ω.

[3] See [17] for an alternate certainty surrogate.

The above conventional dispatch model generally defines a suboptimal forward dispatch in that the generator set-points are *not* optimized to minimize the expected aggregate costs of production [18]. Several authors have advocated a so-called stochastic economic dispatch model, wherein the forward set-points are optimized against the expected real-time cost of balancing (cf. [19–22]). Our option market design can operate in parallel to such an electricity market.

3 A copperplate power system example

We present here a stylized single-bus power system example (adopted from [18]) and illustrate how a bilateral trade (a cash-settled call option) can reduce the volatility in payments of market participants, and even mitigate the risks of financial losses for some. This example serves as a prelude to our centralized call options market design in Section 4.

Consider a power system with two dispatchable generators and a single variable renewable wind power producer serving a demand d. In this example, $\mathbb{G} := \{B, P\}, \quad \text{and} \quad \mathfrak{R} := \{W\}$, where B is a base-load generator, P is a peaker power plant, and W is a wind power producer.

Let $x_B^{\text{cap}} = x_P^{\text{cap}} = \infty, \quad \text{and} \quad \ell_B = 0,\ \ell_P = \infty$. Therefore, B and P have unlimited generation capacities. B does not have the flexibility to alter its output in realtime from its forward set-point. In contrast, P has no ramping limitations. Let B and P have linear costs of production. B has a unit marginal cost, and P has a marginal cost of $1/\rho$, where $\rho \in (0,1]$, i.e., P is more expensive than B.

Encode the uncertainty in available wind in the set

$$\Omega := [\mu - \sqrt{3}\sigma, \mu + \sqrt{3}\sigma] \subset \mathbf{R}_+,$$

and take $\mathbb{P}$ to be the uniform distribution over Ω. That is, available wind is uniform with mean μ and variance σ^2. Scenario $\omega \in \Omega$ defines an available wind capacity of $\bar{x}_r^\omega = \omega$. Further, assume that W produces power at zero cost, and $d \geq \mu + \sqrt{3}\sigma$.

This stylized example is a caricature of electricity markets with deepening penetration of variable renewable wind supply. Base-load generators, specifically nuclear power plants, have limited ramping capabilities. Natural gas based peakers can quickly ramp their power outputs. Utilizing them to balance variability can be costly. Finally, (aggregated) demand is largely inflexible but predictable. In the remainder of this section, we analyze the effect of a bilateral call option on the market outcome for this example.

The conventional dispatch model yields the following forward and realtime dispatch decisions X^*, $x^{*,\omega}$, and the forward and realtime prices $P^*, p^{*,\omega}$, respectively. See [18] for the calculations.

- $X_B^* = d - \mu,\ X_P^* = 0,\ X_W^* = \mu.$
- $x_B^{\omega,*} = d - \mu,\ x_W^{\omega,*} = \min\{\omega, \mu\},\ x_P^{\omega,*} = (\mu - \omega)^+.$
- $P^* = 1,\ p^{\omega,*} = (1/\rho)\mathbb{1}_{\{\omega \in [\mu - \sqrt{3}\sigma, \mu]\}}.$

The dispatch and the prices yield the following payments of the market participants in scenario ω:

$$\pi_B^\omega = d - \mu,\ \ \pi_P^\omega = (\mu - \omega)^+/\rho,\ \ \pi_W^\omega = \mu - (\mu - \omega)^+/\rho.$$

It is straightforward to conclude from the above payments that W incurs a financial loss for scenarios in

$$\Omega_0^- := \Big[\mu - \sqrt{3}\sigma,\ \mu(1-\rho)\Big). \tag{1}$$

when $\rho < \sqrt{3}\frac{\sigma}{\mu}$. In what follows, we consider a bilateral call option trade between P and W, and demonstrate that the option trade reduces the volatilities of P and W's payments, and further shrinks the scenarios in Ω_0^- where W incurs a financial loss.

Adding call option trade between W and P

A cash-settled call option allows its holder the right to claim a monetary reward equal to the positive difference between the real-time price and the strike price of an underlying commodity for an upfront fee. Consider the case where W buys call options from P in the copperplate power system example. We model the bilateral option trade between P and W as a robust Stackelberg game (see [23]) $\mathscr{G}$ as follows. Right after the day ahead market is settled at $t = 0$, P announces an option price $q \in \mathbf{R}_+$ and a strike price $K \in \mathbf{R}_+$ for the call option it sells. Then, W responds by purchasing $\Delta \in \mathbf{R}_+$ options.[4] This option entitles W to a cash payment of $(p^{\omega,*} - K)^+ \Delta$ from P in scenario ω. The option costs W a fee of $q\Delta$. Assume that there is an exogenously defined cap of $\sqrt{3}\sigma$ on the amount of option W can buy from P. The cap equals the maximum shortfall that W can incur from the electricity market in real time.

When they agree on the trade triple (q, K, Δ), the total payments to P and W in scenario ω are given by

$$\begin{aligned} \Pi_W^\omega(q, K, \Delta) &:= \pi_W^\omega - q\Delta + (p^{\omega,*} - K)^+ \Delta, \\ \Pi_P^\omega(q, K, \Delta) &:= \pi_P^\omega + q\Delta - (p^{\omega,*} - K)^+ \Delta, \end{aligned} \tag{2}$$

respectively. In each expression, the first term is the payment from the electricity market, and the other two terms come from the option trade. Assume that W is risk-neutral and has the correct conjectures on the real-time prices. Then, the perceived payoff for W in the day-ahead stage is given by $\mathbb{E}[\Pi_W^\omega(q, K, \Delta)]$.

The possible outcomes of the option trade are identified as the set of Stackelberg equilibria (SE) of $\mathscr{G}$. Precisely, we say $(q^*, K^*, \Delta^*(q^*, K^*))$ constitutes a Stackelberg equilibrium, if

$$\mathbb{E}[\Pi_P^\omega(q^*, K^*, \Delta^*(q^*, K^*))] \geq \mathbb{E}[\Pi_P^\omega(q, K, \Delta^*(q, K))],$$

where $\Delta^* : \mathbf{R}_+^2 \to [0, \sqrt{3}\sigma]$ is the best response of W to the prices $(q, K) \in \mathbf{R}_+^2$ announced by P. For a given (q, K), the best response Δ^* satisfies

$$\mathbb{E}[\Pi_W^\omega(q, K, \Delta^*(q, K))] \geq \mathbb{E}[\Pi_W^\omega(q, K, \Delta(q, K))]$$

for all $\Delta : \mathbf{R}_+^2 \to [0, \sqrt{3}\sigma]$.

Proposition 1. *The Stackelberg equilibria of $\mathscr{G}$ are given by $(q^*, K^*) \in \mathbf{R}_+^2$ and $\Delta^* : \mathbf{R}_+^2 \to [0, \sqrt{3}\sigma]$, that satisfy one of the following conditions:*

- $2q^* + K^* > \rho^{-1}$, *and* $\Delta^* = 0$,
- $2q^* + K^* = \rho^{-1}$.

Over all equilibria with $\Delta^* = \sqrt{3}\sigma$,

$$\mathsf{var}\left[\Pi_i^\omega(q^*, K^*, \sqrt{3}\sigma)\right] - \mathsf{var}\left[\pi_i^\omega\right] = -\frac{3}{2} q^* K^* \sigma^2 < 0$$

for each $i = W, P$.

The first kind of equilibria with $\Delta^* = 0$ describes the degenerate case, where P and W do not participate in the option market. Trade occurs at equilibria of the second kind, and the option/strike prices satisfy $2q^* + K^* = \rho^{-1}$. While we explicitly state the variance reduction only for the case $\Delta^* = \sqrt{3}\sigma$, the variance reduces more generally for any $\Delta^* \neq 0$, as our proof reveals. Lower ρ implies a costlier peaker plant, that in turn indicates a higher price variation in the realtime market. Then, one would expect the potential for volatility reduction (through option trade) to grow with lower ρ. Notice how Proposition 1 reveals the same – smaller ρ leads to equilibria with higher option and strike prices, owing to the relation $2q^* + K^* = \rho^{-1}$, allowing the reduction $\frac{3}{2} q^* K^* \sigma^2$ to increase. The reduction in variance also increases with σ^2. Again, participants stand to gain more (in terms of volatility reduction) from the option trade as the available wind becomes more uncertain.

[4] Allow fractional Δ for simplicity.

Finally, recall that W suffers a financial loss in the energy market whenever $\rho < \sqrt{3}\frac{\sigma}{\mu}$ and $\omega \in \Omega_0^-$ as defined in (1). Here, $\Pi_W^{\omega,*}(q^*, K^*, \Delta^*) < 0$ for $\omega \in \Omega^-$, where

$$\Omega^- := \left[\mu - \sqrt{3}\sigma,\ \mu(1-\rho) - \rho q^* \Delta^*\right).$$

With the bilateral option trade, W suffers a loss only for scenarios in $\Omega^- \subset \Omega_0^-$, i.e., W is less exposed to negative payments with option trade.

Our analysis ignores any incentives market participants may have to strategize their actions in the electricity and the option market together. Characterizing the effects of that co-optimization defines an interesting direction for future work.

Limitations of bilateral trading

The former analysis on the copperplate power system example reveals the benefits of a bilateral trade in call options. In a wholesale market with a collection of dispatchable generators $\mathfrak{G}$ and variable generators $\mathfrak{R}$, one can conceive of $|\mathfrak{G}| \cdot |\mathfrak{R}|$ bilateral option trades. It is difficult to convene and settle such a large number of trades on a regular basis. We propose a centralized clearing mechanism for call option trade to circumvent that difficulty.

4 A Centralized market clearing for cash-settled call options

Consider a market maker M who acts as an aggregate buyer for a collection of option sellers $\mathfrak{G}$, and acts as a seller for the option buyers $\mathfrak{R}$. The SO or a financial institution can fulfill the role of such an intermediary. We now describe the step-by-step procedure for clearing the option market.

Day-ahead stage:

- M broadcasts a set of allowable trades $\mathscr{A}_0$, given by
$$\mathscr{A}_0 := [0, \overline{q}] \times [0, \overline{K}] \times [0, \overline{\Delta}] \subset \mathbf{R}_+^3,$$
to all market participants $\mathfrak{G} \cup \mathfrak{R}$.
- Each $i \in \mathfrak{G} \cup \mathfrak{R}$ submits an acceptable (compact) set of option trades, denoted by $\mathscr{A}_i \subseteq \mathscr{A}_0$.[5]
- M correctly conjectures the real-time prices $p^{\omega,*}$ in each scenario ω, and solves the following stochastic optimization problem to clear the option market.

$$\begin{aligned}
&\text{minimize} \quad \sum_{i \in \mathfrak{G} \cup \mathfrak{R}} \mathsf{var}[\Pi_i^\omega], \\
&\text{subject to} \\
&\quad \sum_{g \in \mathfrak{G}} \Delta_g = \sum_{r \in \mathfrak{R}} \Delta_r, \\
&\quad (q_g, K_g, \Delta_g) \in \mathscr{A}_g,\ (q_r, K_r, \Delta_r) \in \mathscr{A}_r, \\
&\quad \left.\begin{aligned}
&\delta_g^\omega \in [0, \Delta_g], \\
&\sum_{g \in \mathfrak{G}} \delta_g^\omega = \sum_{r \in \mathfrak{R}} \Delta_r \mathbb{1}_{\{p^{\omega,*} \geq K_r\}}, \\
&\mathsf{MS}^\omega = 0,
\end{aligned}\right\} \quad \mathbb{P}-\text{a.s.}, \\
&\quad \text{for each}\ \ g \in \mathfrak{G},\ r \in \mathfrak{R},
\end{aligned} \tag{3}$$

[5]The SO can fix a parametric description of $\mathscr{A}$'s, and market participants report their parameter choices.

where MS^ω is the merchandising surplus (sum of payments) for M in scenario ω, given by

$$\mathsf{MS}^\omega := \sum_{r\in\mathfrak{R}} q_r\Delta_r - \sum_{g\in\mathfrak{G}} q_g\Delta_g - \sum_{r\in\mathfrak{R}} (p^{\omega,*} - K_r)^+ \Delta_r + \sum_{g\in\mathfrak{G}} (p^{\omega,*} - K_g)^+ \delta_g^\omega.$$

The optimization is over $(q_r, K_r, \Delta_r) \in \mathbf{R}_+^3$ for each $r \in \mathfrak{R}$, $(q_g, K_g, \Delta_g) \in \mathbf{R}_+^3$ and $\mathscr{F}$-measurable maps $\delta_g^\omega : \Omega \to [0, \Delta_g]$ for each $g \in \mathfrak{G}$. We let δ_g^ω have a finite energy, i.e., it belongs to the space of $\mathscr{L}_2(\Omega)$ functions.

- Buyer r pays $q_r^*\Delta_r^*$ to M.
- M pays $q_g^*\Delta_g^*$ to seller g.

Real-time stage:

- Scenario ω is realized, and the real-time price of electricity $p^{\omega,*}$ is computed by M.
- M pays $(p^{\omega,*} - K_r^*)^+ \Delta_r^*$ to buyer r.
- Seller g pays $\left(p^{\omega,*} - K_g^*\right)^+ \delta_g^{\omega,*}$ to M.

The constraints in (3) impose that the volume of options bought equals the amount that is sold, all trades are acceptable to market participants, and options cashable in each scenario can be allocated to the sellers. Imposing $\mathsf{MS}^\omega = 0$ ensures that the market maker maintains zero balance from the option trade, and purely facilitates the trade among the market participants. The objective aims at reducing payment volatilities in aggregate among acceptable trades.

How market participant i decides $\mathscr{A}_i$

Consider a seller $g \in \mathfrak{G}$ who expects a revenue π_g^ω in scenario ω. From the electricity and option market, she receives a payoff of

$$\pi_g^\omega + q_g\Delta_g - (p^{\omega,*} - K_g)^+ \delta_g^\omega$$

in scenario ω with the trade triple (q_g, K_g, Δ_g), if M allocates $\delta_g^\omega \in [0, \Delta_g]$. Having no control over δ_g^ω, assume that g conjectures the worst case outcome $\delta_g^\omega = \Delta_g^\omega$ that minimizes her payoff, given by

$$\Pi_g^\omega(q_g, K_g, \Delta_g) := \pi_g^\omega + q_g\Delta_g - (p^{\omega,*} - K_g)^+ \Delta_g.$$

Being risk-neutral, she accepts the trade (q_g, K_g, Δ_g), if

$$\mathbb{E}[\Pi_g^\omega(q_g, K_g, \Delta_g)] \geq \mathbb{E}[\pi_g^\omega].$$

Therefore, $\mathscr{A}_g$ precisely contains the trade triples satisfying the above inequality[6]. For each buyer $r \in \mathfrak{R}$, the set $\mathscr{A}_r$ is defined similarly.

We record salient features of the option market mechanism in the following result.

Proposition 2. *Problem* (3) *admits an optimal solution that satisfies*

$$\sum_{i\in\mathfrak{G}\cup\mathfrak{R}} \mathsf{var}[\Pi_i^{\omega,*}] - \sum_{i\in\mathfrak{G}\cup\mathfrak{R}} \mathsf{var}[\pi_i^\omega] \leq 0.$$

Furthermore, variance in payments of participant $i \in \mathfrak{G}\cup\mathfrak{R}$ reduces if and only if $\mathsf{cov}(A_i^\omega + B_i^\omega, B_i^\omega) < 0$, where

$$A_r^\omega := 2p_r^{\omega,*}(x_r^{\omega,*} - X_r^*),\ B_r^\omega := (p_r^{\omega,*} - K_r)^+\Delta_r,$$
$$A_g^\omega := -2p_g^{\omega,*}(x_g^{\omega,*} - X_g^*),\ B_g^\omega := (p_g^{\omega,*} - K_g)^+\delta_g^\omega.$$

for each $r \in \mathfrak{R}$ and $g \in \mathfrak{G}$.

[6] Risk aversion of g can be modeled by replacing the expectation with a risk-functional (cf. Section 5)

The centralized option trade guarantees a reduction in volatility of payments for market participants in aggregate. Aggregate reduction does not however guarantee the same for each participant. The volatility of a participant reduces when the total payments in realtime (from energy and option markets) are *anti-correlated* with the payments from the option market alone. It is in line with the intuition that variance will decrease when the option market supplements the payments from the energy market.

5 Centralized option trade for the copperplate power system example

We now contrast the outcomes of the centralized option trade on the copperplate power system example in Section 3 with that from the bilateral one. A key observation is that the centralized option trade allows the wind power producer W and the peaker power plant P to derive a reduction in their volatilities that is no less than that obtained from the bilateral trade.

Consider an option market between buyer W and seller P, where the intermediary M chooses a cap on all option prices and volumes a priori. Let the price cap be given by the maximum realtime price $1/\rho$, and the trade volume be capped at $\sqrt{3}\sigma$, the maximum energy shortfall in available wind from its forward contract. Said otherwise, M restricts trade tripes to the set $\mathscr{A}_0 := [0, 1/\rho] \times [0, 1/\rho] \times [0, \sqrt{3}\sigma]$. Then, the set of acceptable trades for P and W are given by

$$\begin{aligned} \mathscr{A}_P &= \{(q_P, K_P, \Delta_P) \in \mathscr{A}_0 : K_P + 2q_P \geq 1/\rho\}, \\ \mathscr{A}_W &= \{(q_W, K_W, \Delta_W) \in \mathscr{A}_0 : K_W + 2q_W \leq 1/\rho\}. \end{aligned} \tag{4}$$

From the above sets, it is straightforward to infer the feasible set of the option market clearing problem in (3), given by $(q_W, K_W, \Delta_W) = (q_P, K_P, \Delta_P) = (q, K, \Delta)$ that satisfies

$$2q + K = 1/\rho, \quad \delta_P^{\omega} = \Delta \mathbb{1}_{\{\omega \leq \mu\}}, \quad \Delta \in [0, \sqrt{3}\sigma].$$

The above trades coincide with the set of all (non-degeraate) Stackelberg equilibria of the bilateral trade between W and P in Proposition 1. Given the objective of the option market clearing problem (3), we conclude that the trade mediated by the market maker finds the equilibrium in the bilateral trade with the highest aggregate variance reduction. We characterize that reduction in the following result.

Proposition 3. *The optimal solutions of* (3) *for the copperplate power system example are given by* $(q_W, K_W, \Delta_W) = (q_P, K_P, \Delta_P) = (q, K, \Delta)$*, where*

$$q = \frac{\sqrt{3}\sigma}{4\rho\Delta}, \quad K = \frac{1}{2\rho} - \frac{\sqrt{3}\sigma}{4\rho\Delta}$$

for each Δ *in* $\left[\sqrt{3}\sigma/2, \sqrt{3}\sigma\right]$*. Moreover, the variance in the payments reduce by* $\frac{3\sigma^2}{16\rho^2}$ *for both* W *and* P*.*

As in the bilateral case, smaller ρ implies higher price variations in the realtime market, leading to greater variance reduction via option trade. We plot the payments of W and P across the scenarios with the parameters in Figure 1. Besides decreasing each player's volatility (at no cost to the intermediary), the diagram reveals how W is less exposed to negative payments than without the option trade. On the other hand, P is now exposed to negative payments in some scenarios. Note that P here is risk-neutral, and we will show later in the paper that if P is risk-averse, she hedges against such losses by requiring more premium q in day-ahead.

The case with two peakers

Consider the case where another peaker plant P' joins the market. Assume P' has infinite capacity with a linear cost of production. Let its marginal cost be higher than that of P. Then, P' will never be dispatched in the conventional market, and hence, does not get paid from the electricity market, but participates in the option trade as a seller. Earnings of W remain unaffected. Options bought by W are split between P and P'. Volatility in P's payment still reduces, albeit to a lesser extent than without P' in the market. This analysis illustrates how peakers that are rarely necessary

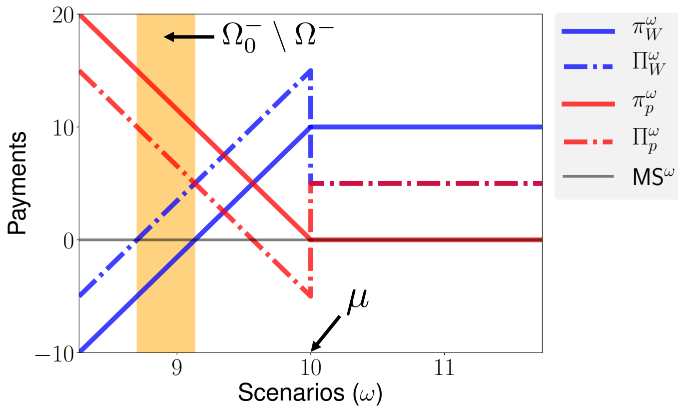

Figure 1: Payments of the market participants and the merchandising surplus for the copperplate power system example with $\mu = 10, \sigma^2 = 1$ and $\rho = \frac{\sqrt{3}}{20}$. Option trade allows W to avoid financial loss in the shaded set of scenarios $\Omega_0^- \setminus \Omega^-$.

to produce but important for resource adequacy can rely on the option market for a steady remuneration. Flexible ramping products introduced in CAISO and MISO's markets have a similar goal – it compensates peakers to remain online to guarantee resource adequacy during large wind ramps (cf. [24]). While such products amount to SO 'buying' capacity from the peakers, our mechanism is non-binding and financially incentivizes them to stay and participate in the electricity market.

When the market participants are risk-averse

In Section 4, we defined the set of acceptable trades of market participants, assuming them to be risk-neutral. Any trade was deemed acceptable as long as the *expected* payment from the electricity market improves by participating in the option market. Evidence from electricity markets suggest that participants are often risk averse, e.g., see [11, 25]. In this vein, we study the effect of risk aversion on the set of acceptable trades.

Assume that a market participant perceives risk via the *conditional value at risk* functional (cf. [26]), and finds a trade triple (q, K, Δ) acceptable, if

$$\mathsf{CVaR}_{\alpha_i}[-\Pi_i^\omega(q, K, \Delta)] \leq \mathsf{CVaR}_{\alpha_i}[-\pi_i^\omega], \tag{5}$$

where π_i^ω and Π_i^ω describe its payments from the energy market and the energy-cum-option market in scenario ω. The CVaR risk measure is given by

$$\mathsf{CVaR}_\alpha[z^\omega] := \min_{t \in \mathbf{R}} \left\{ t + \frac{1}{1-\alpha} \mathbb{E}[(z^\omega - t)^+] \right\}$$

for an $\mathscr{F}$-measurable map z. Parameter $\alpha \in [0, 1)$ encodes the extent of risk-aversion. If z^ω is the monetary loss in scenario ω, $\mathsf{CVaR}_\alpha[z^\omega]$ equals the expected loss over the α% scenarios that result in the highest losses.

To study the effect of risk-aversion, we plot the boundaries of $\mathscr{A}_W$ and $\mathscr{A}_P$ – the sets of acceptable trades for the wind power producer and the peaker power plant in our copperplate power system example, respectively – for various values of α_W and α_P in Figure 2.[7] Acceptable trades at each α for W lie to the left of the corresponding surface. For P, they lie to the right of it. The surfaces with $\alpha = 0$ correspond to risk-neutral players. In that case, linearity of expectation allows one to deduce that the acceptability of a trade is independent of the number of options Δ. As a result, the surfaces for the risk-neutral case are vertical planes. That independence no longer holds for risk-averse players, and the surfaces lose planarity. As α grows, P requires higher forward premium (option price q_P) for a given volume Δ_P. Similar conclusions hold for W. She becomes less willing to accept trades with a higher forward premium, the more risk-averse she gets.

[7] The current diagram stands as a correction to [27, Figure 2].

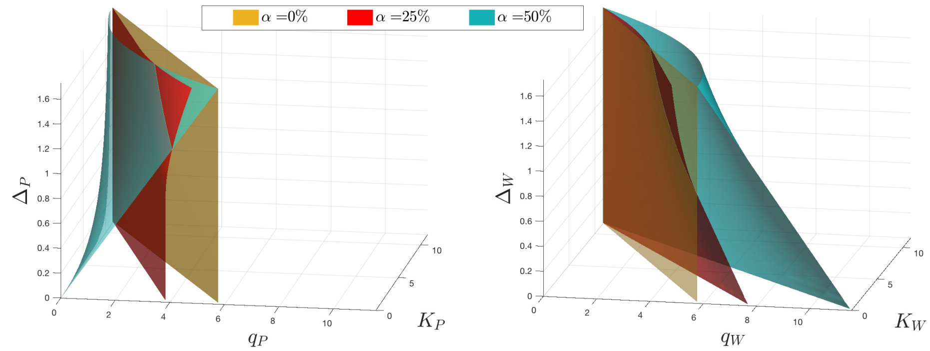

Figure 2: The boundaries of $\mathscr{A}_P$ and $\mathscr{A}_W$ are portrayed respectively on the left and the right, for the copperplate power system example in Section 5, when P and W both measure risk via CVaR_α for various values of α's. In our experiments, we assume $\mu = 10, \sigma^2 = 1$ and $\rho = \frac{\sqrt{3}}{20}$, and $\Delta \in [0, \sqrt{3}]$, and compute the sets via the technique outlined in [28, equation (6)].

6 Option market for the IEEE 14-bus test system

We now explore the outcomes from the electricity and option market on a modified IEEE 14-bus test system shown in Figure 3. Relevant data is adopted from MATPOWER [29]. Two wind power producers are added to the network at buses 6 and 14, each with a uniformly distributed available wind with mean 50MW. All transmission lines are assumed to have a capacity of 35MW, except that between buses 1 and 2 (20 MW) and another between buses 2 and 4 (20MW). We adopt the so-called linear *DC power flow* model that sets voltage magnitudes to their nominal values, neglects line resistances, and deems voltage phase angle differences to be small.

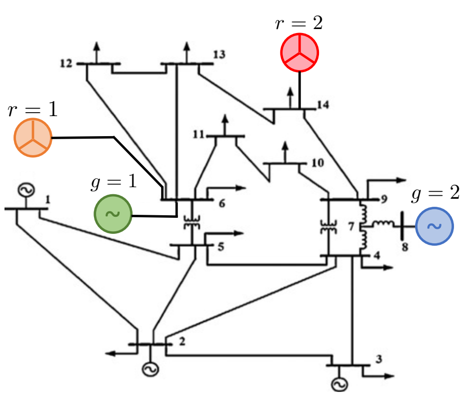

Figure 3: One line diagram of the IEEE 14-bus test system with wind generators added to buses 6 and 14. We consider an option market between buyers $r = 1, 2$ and sellers $g = 1, 2$.

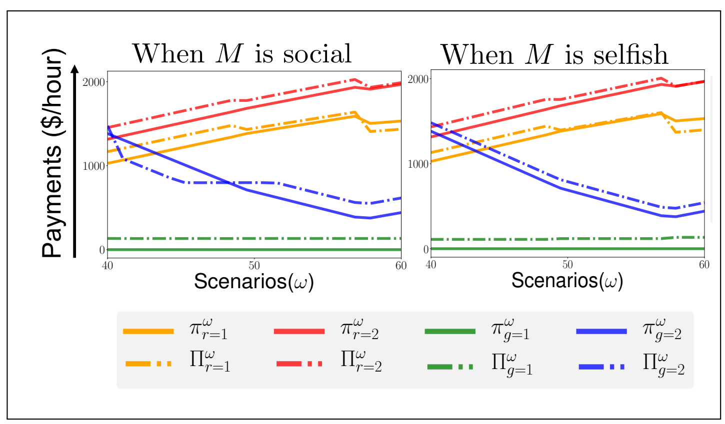

Figure 4: Payments to the market participants in the IEEE 14-bus system with and without the option market trade. The figure on the left considers a social market maker, while the one on the right is derived with a profit-maximizing one.

Consider an option market with the wind power producers at buses 6 and 14 as buyers, and the dispatchable generators at buses 6 and 8 as sellers. The sellers are generators with higher production costs compared to others in the power system. Assume zero production costs for the wind generators. In our experiments, we use $\bar{\Delta} = 10$MW, and vary the available wind between 40 and 60 MWs. The market clearing procedure is implemented as a Jupyter notebook at [15].[8]

[8]The problem in (3) is nonconvex and nonlinear, that we solve using sequential least-squares quadratic programming [30]. Nonsmooth functions $\mathbb{1}_{\{x \geq 0\}}$ and $(x)^+$ are replaced by their smooth surrogates $(1 + e^{-\alpha x})^{-1}$ and $x(1 + e^{-\alpha x})^{-1}$, respectively for a sufficiently large α.

Figure 4 plots the payments of the market participants with and without the option market. Seller $g = 1$ at bus 6 is never dispatched, and hence, receives no payment in the electricity market. Option trade provides that seller a remuneration, thus incentivizing it to remain in the market. Its variance in payments remains at zero, however. On the contrary, the option market reduces the variance for seller $g = 2$ by nearly 58%.

When the market maker is a profit-maximizer

The option market mechanism in (3) assumes a social intermediary. Next, consider a selfish market maker that aims at maximizing its expected merchandising surplus, and solves

$$\begin{aligned}
&\text{maximize} \quad \mathbb{E}[\mathsf{MS}^{\omega}],\\
&\text{subject to}\\
&\qquad \sum_{g\in\mathfrak{G}} \Delta_g = \sum_{r\in\mathfrak{R}} \Delta_r,\\
&\qquad (q_g, K_g, \Delta_g) \in \mathscr{A}_g,\ (q_r, K_r, \Delta_r) \in \mathscr{A}_r,\\
&\qquad \left.\begin{aligned} &\delta_g^{\omega} \in [0, \Delta_g]\\ &\sum_{g\in\mathfrak{G}} \delta_g^{\omega} = \sum_{r\in\mathfrak{R}} \Delta_r \mathbb{1}_{\{p^{\omega,*} \geq K_r\}} \end{aligned}\right\} \quad \mathbb{P}-\text{a.s.},\\
&\qquad \text{for each } \ g \in \mathfrak{G},\ r \in \mathfrak{R}.
\end{aligned} \tag{6}$$

In our stylized example, the above scheme reduces the variance in payments of the market participants; see [27]. The reductions are larger, however, with a social intermediary. The market outcomes for the IEEE 14-bus test system example tells a similar story (cf. Figure 4). Only variance reduction for $g = 2$ with a social market maker disappears under a selfish one. Although motivated by maximizing profit, our final result in Proposition 4 proves that a selfish intermediary does not make any, on average!

Proposition 4. $\mathbb{E}[\mathsf{MS}^{\omega,*}] = 0$ *at an optimal solution of (6).*

Hedging against spatial price risks

Call options are instruments to hedge against temporal price variations. One can also hedge against *spatial* price variations using instruments such as Financial Transmission Rights (FTRs) [31]. Holding f FTRs between buses a and b entitles a market participant to receive a payment of

$$\mathsf{FTR}^{\omega}(a, b, f) := (p_b^{\omega,*} - p_a^{\omega,*}) f$$

in scenario ω. Thus, an option buyer $r \in \mathfrak{R}$ located at bus b holding an FTR between buses a and b receives a total payment of

$$\Pi_r^{\omega} = \pi_r^{\omega} + \mathsf{FTR}^{\omega}(a, b, f) - q_r^* \Delta_r^* + \left(p_b^{\omega,*} - K_r^*\right)^+ \Delta_r^*.$$

Figure 5 illustrates how $r = 2$ can reduce its volatility in payments by holding $f = 20$MW worth of FTR's between buses 9 and 14, in addition to the reduction it attains from the option trade.

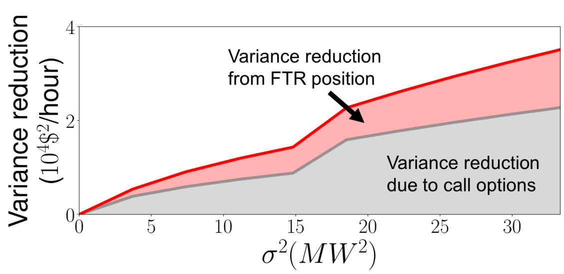

Figure 5: Variance reduction in the payment of $r = 2$ as a function of σ^2 in the IEEE 14-bus test system. Holding an FTR between buses 9 and 14 significantly increases the reduction.

7 Concluding remarks

Price volatility in electricity markets is an inevitable consequence of integrating large scale wind energy. In this paper, we proposed a centralized market for call options for market participants to tackle the attending financial risks. The centralized mechanism (mediated by a market maker) generalizes bilateral trading of call options. On a stylized copperplate power system example, this market provably reduces the payment volatilities of market participants. Numerical experiments on an IEEE 14-bus test system also appear encouraging. This preliminary work provides the foundation for a number of future research endeavors. For example, this paper did not model the possibility that market participants can strategize between energy and option market offers. Effect of such strategic interactions on energy market efficiency is important to characterize. For adoption in practice, one also needs to estimate the trade volume with real market data from regions with high wind penetration (e.g., Germany, Texas, Denmark). Finally, operating such an option market in conjunction with current electricity markets will require a carefully designed legal and regulatory framework.

8 Appendix

Proof of Proposition 1

Let P choose $(q,K) \in \mathbf{R}_+^2$. Then, W's payoff from the option trade alone is given by

$$V_W^\omega(q,K,\Delta) := \Pi_W^\omega(q,K,\Delta) - \pi_W^\omega, \tag{7}$$

that upon utilizing (2) yields

$$\mathbb{E}[V_W^\omega(q,K,\Delta)] = \begin{cases} -q\Delta, & \text{if } K > 1/\rho, \\ -\frac{\Delta}{2}\left(2q+K-\rho^{-1}\right), & \text{otherwise.} \end{cases}$$

We now describe W's best response to P's action.

- If $2q+K < \rho^{-1}$, then W responds by playing $\Delta = \sqrt{3}\sigma$.
- If $2q+K = \rho^{-1}$, then W is agnostic to Δ in $[0, \sqrt{3}\sigma]$.
- If $2q+K > \rho^{-1}$, then W chooses $\Delta = 0$.

Define $V_P^\omega(q,K,\Delta) := \Pi_P^\omega(q,K,\Delta) - \pi_P^\omega$, as the payoff of P from the option trade. Then, the relation in (2) yields

$$\mathbb{E}[V_P^\omega(q,K,\Delta)] = -\mathbb{E}[V_W^\omega(q,K,\Delta)]. \tag{8}$$

Given W's choices, we have the following cases.

- If $2q+K < \rho^{-1}$, then $\mathbb{E}[V_P^\omega(q,K,\Delta)] < 0$. Therefore, P will avoid playing such a (q,K).
- If $2q+K = \rho^{-1}$, then $\mathbb{E}[V_P^\omega(q,K,\Delta)] = 0$, and P is agnostic to W's choice of Δ in $[0, \sqrt{3}\sigma]$.
- If $2q+K > \rho^{-1}$, then W responds with $\Delta = 0$. And, P receives zero income from option trade.

Combining them yields the equilibria of $\mathscr{G}$. Now, the difference in variances for W with and without the option trade can be shown to equal

$$2\mathsf{cov}(\pi_W^\omega, V_W^{\omega,*}) + \mathsf{var}\left[V_W^{\omega,*}\right]. \tag{9}$$

When $2q^* + K^* = \rho^{-1}$, we have

$$V_W^{\omega,*}(q^*,K^*,\Delta^*) = \begin{cases} q^*\Delta^*, & \text{if } \omega \le \mu, \\ -q^*\Delta^*, & \text{otherwise.} \end{cases}$$

Utilizing $\pi_W^\omega = \mu - (\mu-\omega)^+/\rho$ and $V_W^{\omega,*}$ from the above relation in (9), we conclude

$$\begin{aligned}
&\mathsf{var}\left[\Pi_W^\omega(q^*,K^*,\Delta^*(q^*,K^*))\right] - \mathsf{var}\left[\pi_W^\omega\right] \\
&\quad = -(2/\rho)\mathsf{cov}((\mu-\omega)\mathbb{1}_{\{\omega<\mu\}}, V_W^{\omega,*}) + \mathsf{var}\left[V_W^{\omega,*}\right] \\
&\quad = -\frac{1}{\rho\sqrt{3}\sigma}\int_{\mu-\sqrt{3}\sigma}^{\mu}(\mu-\omega)q^*\Delta^* d\omega + (q^*\Delta^*)^2 \\
&\quad = -\frac{q^*\Delta^*\sqrt{3}\sigma}{2\rho} + (q^*\Delta^*)^2 \\
&\quad = -q^*\Delta^*\sqrt{3}\sigma\left(q^* + \frac{K^*}{2}\right) + (q^*\Delta^*)^2 \\
&\quad = (q^*)^2\Delta^*(\Delta^* - \sqrt{3}\sigma) - q^*K^*\Delta^*\sqrt{3}\sigma/2.
\end{aligned} \tag{10}$$

The last expression is nonpositive because $\Delta^* \in [0,\sqrt{3}\sigma]$. For P, we have $\pi_P^\omega = \mu - \pi_W^\omega$ and hence, $V_P^{\omega,*} = -V_W^{\omega,*}$. Therefore, the variance reduction in P's payments equals that in W's. The rest follows from substituting $\Delta^* = \sqrt{3}\sigma$ in (10).

Proof of Proposition 2

Recall that Ω is compact and $\delta_g \in \mathscr{L}_2(\Omega)$ for each g. The constraint set of (3) is then compact. An appeal to Weierstrass Theorem [32] guarantees the existence of an optimum, given the continuity of the objective function. All Δ's being zero constitutes a feasible point of (3), where the objective function is zero. The minimum of that objective is therefore nonpositive.

Next, from (7) and (9), we have

$$\begin{aligned}
&\mathsf{var}\left[\Pi_r^\omega\right] - \mathsf{var}\left[\pi_r^\omega\right] \\
&\quad = 2\mathsf{cov}(\pi_r^\omega, V_r^\omega) + \mathsf{var}\left[V_r^\omega\right] \\
&\quad = \mathsf{cov}(2[P^*X_r^* + p^{\omega,*}(x_r^{\omega,*} - X_r^*)] + V_r^\omega, V_r^\omega) \\
&\quad = \mathsf{cov}(2p^{\omega,*}(x_r^{\omega,*} - X_r^*) + (p_r^{\omega,*} - K_r)^+\Delta_r, \\
&\qquad\qquad (p_r^{\omega,*} - K_r)^+\Delta_r) \\
&\quad = \mathsf{cov}(A_r^\omega + B_r^\omega, B_r^\omega)
\end{aligned}$$

for each $r \in \mathfrak{R}$. The argument for $g \in \mathfrak{G}$ is similar and omitted for brevity.

Proof of Proposition 3

The feasible set of (3) for the copperplate power system example coincides with the set of nontrivial equilibria of the bilateral trade. We conclude from (10) in the proof of Proposition 1 that (3) amounts to solving

$$\begin{aligned}
&\text{minimize} \quad 2q^2\Delta(\Delta - \sqrt{3}\sigma) - qK\Delta\sqrt{3}\sigma, \\
&\text{subject to} \quad 2q + K = \rho^{-1}, q \geq 0, K \geq 0, \\
&\qquad\qquad\quad\; 0 \leq \Delta \leq \sqrt{3}\sigma.
\end{aligned} \tag{11}$$

Replacing $K = \rho^{-1} - 2q$, the objective function of the above problem simplifies to $2q^2\Delta^2 - q\Delta\sqrt{3}\sigma\rho^{-1}$. Being convex quadratic in q, it is minimized at

$$q^*(\Delta) = \min\left\{\frac{\sqrt{3}\sigma}{4\rho\Delta}, \frac{1}{2\rho}\right\}$$

for each $\Delta \in [0,\sqrt{3}\sigma]$. Split the analysis into two cases:

- Case $\Delta \leq \frac{\sqrt{3}}{2}\sigma$: Then, $q^*(\Delta) = \frac{1}{2\rho}$, and the objective function of (11) simplifies to $\frac{1}{2\rho^2}\Delta(\Delta - \sqrt{3}\sigma)$. That function is minimized at $\Delta = \frac{\sqrt{3}}{2}\sigma$, taking the value $-\frac{3\sigma^2}{8\rho^2}$.
- Case $\Delta > \frac{\sqrt{3}}{2}\sigma$: Then, we have $q^*(\Delta) = \frac{\sqrt{3}\sigma}{4\rho\Delta}$ for each Δ, for which the objective function of (11) further simplifies to a constant $-\frac{3\sigma^2}{8\rho^2}$.

Combining the above two cases and computing the variance reduction at the outcome yields the required result.

Proof of Proposition 4

Existence of an optimal solution follows along the same lines as in Proposition 2. The definition of $\mathscr{A}_g$, $\mathscr{A}_r$ then yields

$$\mathbb{E}[\Pi_g^\omega] - \mathbb{E}[\pi_g^\omega] \geq 0, \quad \mathbb{E}[\Pi_r^\omega] - \mathbb{E}[\pi_r^\omega] \geq 0$$

for each $g \in \mathfrak{G}$ and $r \in \mathfrak{R}$. Summing the above inequalities over all g and r yields $\mathbb{E}[\mathsf{MS}^\omega] \leq 0$. Furthermore, $\mathbb{E}[\mathsf{MS}^\omega] = 0$ is achieved at a feasible point with all Δ's being identically zero. That completes the proof.

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